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# ON THE EFFECT OF ELECTRON DENSITY INHOMOGENEITIES ON MEASUREMENT OF SOME PARAMETERS OF THE IONOSPHERE

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# ON THE EFFECT OF ELECTRON DENSITY INHOMOGENEITIES ON MEASUREMENT OF SOME PARAMETERS OF THE IONOSPHERE

(O vliyanii neodnorodnostey elektronnoy kontsentratsii ionosfery na izemereniye nekotorykh yeye paranetrov )

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## ABSTRACT

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The method is based on measurement of phase invariant of a three-harmonic wave and it has been proposed for electron density measurement using artificial satellites. The effect of radio diffraction of waves in ionospheric inhomogeneities on the fluctuation of the gas invariant is examined. Discussed is the question of fluctuations of ~~the~~ reduced/ Doppler frequency difference on account of diffraction in random electron concentration inhomogeneities. Estimate is given of the precision of measurement by that method.

## COVER-TO-COVER TRANSLATION

### 1. Phase Invariant Method

Some of the methods of ionosphere and interplanetary gas study with the aid of artificial satellites and rockets have been considered in a series of works [1—3]. In particular, there is question of measurement of electron concentration $N(z_0)$ at satellite's position and of the integral electron concentration along the Earth's radius — $\int_0^{z_0} N(z)dz$. Various effects may be used for the measurement of these parameters of the ionosphere: Doppler frequency shift, effects linked with phase course variations and group time lag in the ionosphere etc.. One thing is common to these methods, and that is the exclusion of effects linked only with the variation of geometrical parameters. It is shown in reference [3] that the proposed methods lead to identical functional dependences of the measured quantities on concentration of electrons.

It is proposed in the current work to use for the determination of $N(z)$ the variation of the phase invariant $\bar{\theta}$ in a three-harmonic wave [4]. The latter method, while giving in substance the same results as the methods examined in [1 - 3], is simpler from the standpoint of the estimate of fluctuations.

In the same approximation as in [3], the phase invariant for a plane three-harmonic wave propagating in the ionosphere, may be written in the form:

$$\bar{\theta} = \varphi_0 - \frac{\varphi_1 + \varphi_2}{2} = \frac{2\pi e^2 \Delta\omega^2}{mc \cos\chi\, \omega_0 \omega_1 \omega_2} \left[ \int_0^{z_0} N(z)\, dz - \right. \tag{1}$$

$$\left. - \operatorname{tg}^2\chi \int_0^{z_0} \frac{N(z)\, z}{R_0}\, dz \right].$$

When deriving this expression, we utilized the expression for the phase course in a spherically-stratified ionosphere [3]:

...

$$L_\Phi = r_0 - \frac{2\pi e^2}{m\omega \cos\chi}\left[\int_0^{z_0} \frac{N}{\omega \pm \omega_L} dz - \mathrm{tg}^2\chi \int_0^{z_0} \frac{Nz}{R_0(\omega \pm \omega_L)} dz\right] \qquad (2)$$

and we considered that $\omega_L = (eH_0/mc)\cos\gamma \ll \omega_0;\ \omega_1;\ \omega_2.$ The following designations were used in the above-presented expressions: $\varphi_0$ for the phase of the carrier frequency $\omega_0$ ; $\varphi_1$, $\varphi_2$ for the oscillation phases of $\omega_1$, $\omega_2$ ; $\chi$ for the zenithal angle of the satellite. The frequencies in the three-harmonic wave are linked by the correlation:

$$\omega_0 - \omega_1 = \omega_2 - \omega_0 = \Delta\omega.$$

The variation of the phase invariant at three-harmonic wave propagation in the medium is conditioned by the dispersion. In the ionosphere dispersion is caused by refractive index dependence on frequency as well as by diffraction in random inhomogeneities of the refractive index. The first cause leads to regular invariant phase variations as a function of distance traveled by the wave in the medium. Diffraction in random inhomogeneities provokes fluctuations of the phase invariant [6, 7].

It is necessary to render more precise the ascertaining of the place of the proposed method in the series of other methods described in literature. By comparison with the method utilizing the Faraday effect, the methods based on the measurement of the Doppler shift of the reduced difference of two coherent frequencies, on the difference of group time lag and of that of phase invariant, are more precise, for their accuracy is not influenced by satellite rotation and geomagnetic inhomogeneity.

In the method utilizing the Doppler frequency differences, measured is the quantity

$$\frac{d\Phi}{dt} \backsim \left[ N(z_0)\left(1 - \frac{z_0}{R_0}\mathrm{tg}^2\chi\right)\frac{dz_0}{dt} + \right.$$

$$\left. + \left( \int_0^{z_0} N\,dz - \frac{3\sec\chi - 1}{R_0}\int_0^{z_0} Nz\,dz \right)\mathrm{tg}\,\chi\,\frac{d\chi}{dt} \right],$$

which differs from $\bar{\theta}(N)$ [1]. Simultaneous measurement of $d\Phi/dt$ and $\bar{\theta}(N)$ (group lag time), all other conditions being identical, will allow a more accurate determination of $N(z_0)$ and $\int_0^{z_0} N dz$.

## 2. Fluctuations of the Phase Invariant

The effect of measured quantities' fluctuations has not been taken into account in all works known to us. It appears to be necessary though to effect this accounting from the standpoint of the effect of fluctuations on measurement precision, as well as with the object of studying the statistical structure of the ionosphere. As was already mentioned, the estimate of phase invariant fluctuations may be made in a comparatively simple way.

To estimate the intensity of fluctuations of the phase invariant of a three-harmonic wave propagating in the ionosphere, it is necessary to find the spectrum of the structural function of the phase invariant. The spectrum of $F_\theta(\varkappa, 0)$ for the case of an arbitrary value of the difference of wave numbers $\Delta k = k_0 - k_1 = k_2 - k_0$: may be obtained by the same method as that used in [5, 6]:

$$F_\Theta(\varkappa, 0) = \frac{4\pi^3 e^4 L}{m^2}\left\{\frac{2}{c^2}\left(\frac{\Delta\omega^4}{\omega_0^2\omega_1^2\omega_2^2}\right) - \frac{k_0^2}{\omega_0^4}\left[1 - \frac{\sin(\varkappa^2 L/k_0)}{\varkappa^2 L/k_0}\right] - \right.$$
$$- \frac{k_1^2}{4\omega_1^2}\left[1 - \frac{\sin(\varkappa^2 L/k_1)}{\varkappa^2 L/k_1}\right] - \frac{k_2^2}{4\omega_2^2}\left[1 - \frac{\sin(\varkappa^2 L/k_2)}{\varkappa^2 L/k_2}\right] +$$
$$+ \frac{k_0 k_1}{\omega_0^2\omega_1^2}\left[1 - \frac{\sin[\varkappa^2(k_0 + k_1)L/2k_0k_1]}{\varkappa^2(k_0 + k_1)L/2k_0k_1}\right] + \frac{k_0k_1}{\omega_0^2\omega_1^2}\left[1 - \frac{\sin(\varkappa^2\Delta k L/2k_0k_1)}{\varkappa^2\Delta k L/2k_0k_1}\right] +$$
$$+ \frac{k_0 k_2}{\omega_0^2\omega_2^2}\left[1 - \frac{\sin[\varkappa^2(k_0 + k_2)L/2k_0k_2]}{\varkappa^2(k_0 + k_2)L/2k_0k_2}\right] + \frac{k_0k_2}{\omega_0^2\omega_2^2}\left[1 - \frac{\sin(\varkappa^2\Delta k L/2k_0k_2)}{\varkappa^2\Delta k L/2k_0k_2}\right] - \tag{3}$$
$$- \frac{k_1 k_2}{2\omega_1^2\omega_2^2}\left[1 - \frac{\sin[\varkappa^2(k_1 + k_2)L/2k_1k_2]}{\varkappa^2(k_1 + k_2)L/2k_1k_2}\right] - \frac{k_1k_2}{2\omega_1^2\omega_2^2}\times$$
$$\left.\times\left[1 - \frac{\sin(\varkappa^2\Delta k L/k_1k_2)}{\varkappa^2\Delta k L/k_1k_2}\right]\right\}\Phi_N(\varkappa, 0),$$

where L is the path over which the three-harmonic wave travels in the medium, and $\Phi_N(\varkappa, 0)$ is the three-dimensional fluctuation spectrum of electron concentration.

It may be seen from the above expression that if the field of fluctuations of electron concentration is locally-uniform and isotropic, the field of fluctuations of the phase invariant (contrary to troposphere [6]) is also locally-uniform. The indicated distinction from the troposphere case is linked with the dispersion of the index of refraction. As was shown in [8], there are at present direct demonstrations of the existence of turbulence in the lower ionosphere. Generally speaking, turbulence in the E-region is not isotropic, while in the F-region the character of inhomogeneities apparently differs substantially from the locally-uniform turbulence. However, for the estimate of the order of magnitude of fluctuations, we shall consider that

$$\Phi_N(\varkappa, 0) = 0{,}033\, C_N^2\, \varkappa^{-11/3} \qquad (\varkappa_0 < \varkappa < \varkappa_{\text{max.}}). \tag{4}$$

Estimates of $C_N^2$ from the analysis of various experimental data [8] lead to the value $C_N^2 \simeq 2.3 \cdot 10^4 \text{ cm}^{-20/3}$.

The inhomogeneity of the field of fluctuations of the phase invariant, linked with the effect of large-scale inhomogeneities, is determined by the effects allowed for by geometrical optics. In substance, these effects may be related to the quantity being measured, and they are automatically taken into account by formula (1). It follows from the expression for the spectrum (3) that the conditions of applicability of geometrical optics has the form and the intensity of "small-scale" fluctuations is determined by the energy included within the limits from $\varkappa_1 \ll 2\pi(\Delta\lambda L)^{-1/2}$ to $\varkappa_{max}$. The concrete selection of the value $\varkappa_1$ has no effect on the final results.

The intensity of "small-scale" fluctuations is expressed by the integral

$$\overline{\Delta\Theta^2} = \int_{\varkappa_1}^{\varkappa_{max}} F_\Theta(\varkappa, 0)\, \varkappa\, d\varkappa. \tag{5}$$

After a series of transformations we may obtain an expression convenient for estimates

$$\overline{\Delta\Theta^2} \doteq \frac{1{,}8\, C_N^2\, \Delta\omega\, \omega_0^2\, \pi^6\, e^8}{m^4\, c\, \omega_1^3\, \omega_2^3}\, \lambda_0^{-1/6} L^{11/6}. \tag{6}$$

The mean value of the phase invariant (measured value) is equal by the order of magnitude to

$$\bar{\Theta} \sim \frac{2\pi e^2 \Delta\omega^2}{m c\, \omega_0 \omega_1 \omega_2} \bar{N} L. \tag{7}$$

The relation

$$K = \frac{\overline{\Theta}}{\sqrt{\overline{\Delta\Theta^2}}} = \frac{4{,}5\,\overline{m}N\,\Delta\,\omega^{3/2}\,\omega_1^{1/2}\,\omega_2^{1/2}}{\pi^3 c^{1/2}\,C_N\,\omega_0^2}(\lambda_0 L)^{1/12} \tag{8}$$

characterises the excess of the measured value over fluctuations. Compiled are in Table 1 the values of this relation for certain wavelengths ($\lambda = 30$ м, $3$ м, $0{,}3$ м) and for distances from 200 to 2000 km.

TABLE 1

| | | λ | | |
|---|---|---|---|---|
| | | 30 м | 3 м | 0,3 м |
| $\Delta\omega \ll \omega_0$ | $\overline{\Theta}$ | $1{,}7(1{,}0 \div 10)$ | $1{,}7(10^{-1} \div 1{,}0)$, | $1{,}7(10^{-2} \div 10^{-1})$ |
| | $K$ | 4,0 | 13,5 | 39,0 |
| $\Delta\omega = \omega_0/2$ | $\overline{\Theta}$ | $1{,}7(10^2 \div 10^3)$ | $1{,}7(10 \div 10^2)$ | $1{,}7(1{,}0 \div 10)$ |
| | $K$ | 1,0 | 4,0 | 11,0 |

It may be seen from that table that for a specific selection of apparatus' parameters one may realize the case, when the regular variations of the phase invariant do not exceed $2\pi$, and the measured quantity $\overline{\Theta}$ is by one order higher than the level of fluctuations. As was already reminded above, the effect of the F-layer was not taken into account in our estimates, and that is why the estimates brought out in Table 1, give only the orders of magnitudes. It is obvious that the effect of the upper ionosphere layers may only lead to the rise of $\overline{\Delta\Theta^2}$. As to the relation K, the form of its dependence on L may vary substantially.

For the construction of apparatus designed for measurement of the phase invariant in a three-harmonic wave, it is necessary to note, that the phase difference of two oscillations, of which one is obtained during the shift of carrier and lateral oscillations, and the other at the shift of the carrier and the other lateral oscillation (also a difference frequency), is a quantity proportional to the phase invariant.

### 3. Fluctuations of the Reduced Difference of Doppler Frequencies

Let us consider the question of the effect of turbulent inhomogemeities on the measurement precision of the Doppler shift of the difference of two coherent frequencies.

The structural function of the reduced difference of phases has the form:

$$D_s(\rho) = \overline{\{[m\varphi_1(\rho, L) - n\varphi_2(\rho, L)] - [m\,\varphi_1(0, L) - n\,\varphi_2(0, L)]\}^2} = \\ = m^2\,\overline{\Delta\varphi_1^2} - 2\,mn\,\overline{\Delta\varphi_1\Delta\varphi_2} + n^2\overline{\Delta\varphi_2^2}, \tag{9}$$

where

$$\overline{\Delta\varphi_{1,2}^2} = \overline{[\varphi_{1,2}(\rho, L) - \varphi_{1,2}(0,L)]^2}\,;$$

$$\overline{\Delta\varphi_1\Delta\varphi_2} = \overline{[\varphi_1(\rho, L) - \varphi_1(0, L)]\,[\varphi_2(\rho, L) - \varphi_2(0, L)]^2},$$

$\varphi_{1,2}(\rho, L)$ being the phase fluctuation in the plane $x = L$ at a distance from the axis x, and $m\omega_1 = n\omega_2 = \omega$.

The two-dimensional spectrum of the phase reduced difference has the form:

$$F_s(\varkappa, 0) = \frac{4\pi^3 e^2 L k^2}{m^2} \left\{ 2\left(\frac{\omega_1^2 - \omega_2^2}{\omega_1^2 \omega_2^2}\right)^2 - \frac{1}{\omega_1^4}\left[1 - \frac{\sin(\varkappa^2 L/k_1)}{\varkappa^2 L/k_1}\right] - \right.$$

$$- \frac{1}{\omega_2^2}\left[1 - \frac{\sin(\varkappa^2 L/k_2)}{\varkappa^2 L/k_2}\right] + \frac{2}{\omega_1^2 \omega_2^2}\left[1 - \frac{\sin[\varkappa^2(k_1 + k_2) L/2 k_1 k_2]}{\varkappa^2(k_1 + k_2)L/2k_1k_2}\right] + \tag{10}$$

$$\left. + \frac{2}{\omega_1^2 \omega_2^2}\left[1 - \frac{\sin[\varkappa^2(k_2 - k_1)L/2 k_1 k_2]}{\varkappa^2(k_2 - k_1)L/2k_1k_2}\right]\right\} \Phi_N(\varkappa, 0).$$

In the use of the coherent frequency method for the determination of electron concentration in the ionosphere, one generally measures the derivative in time from the reduced difference of phases, for the reduced phase difference itself is not single-valued (it varies within limits much greater than $2\pi$). To compute fluctuation of that quantity it is necessary to know the time spectrum of reduced difference of phases.. The simplest way to find the temporal fluctuations is to make use of the hypothesis of "frozen turbulence". The time spectrum of the reduced difference of phases may be found from the spatial spectrum. When utilizing the hypothesis of "frozen turbulence" it is necessary to also account for drift of inhomogeneities in the ionosphere and the motion velocity of the satellite in the direction perpendicular to the line stallite — observer. Because satellite velocity greatly exceeds that of inhomogeneities' drift in the ionosphere, mainly the fluctautions related to satellite's motion will have effect. At the same time, the problem is considered in the same approximation as in [5], i.e. there is question of flat (plane) wave propagation in an infinite turbulent medium.

The fluctuations of the reduced difference of phases lead to the blurring of the spectral line of the reduced Doppler frequency difference.

The problem of interest to us on the measurement precision of the Doppler shift of coherent frequencies' reduced difference is equivalent to the problem of study of generator's spectral line, blurred on account of fluctuation frequency [9, 10]. Thus, there is question of the shape of oscillation's spectral line of the form

$$y(t) = A\cos[\nu_g t + \alpha(t)]; \quad \alpha(t) = \int_{t_0}^{t} \Delta\nu(\xi)d\xi, \tag{11}$$

where A is the amplitude, $\nu_g$ is the measured reduced difference of frequencies, $\alpha(t)$ is the random phase, $\Delta\nu(t)$ — frequency fluctuations. Generally speaking, the process $\alpha(t)$ is not stationary on account of field inhomogeneity of the reduced difference of phases. Nevertheless, if one takes into account the finiteness of observation time, or reduces the frequencies near zero, as this was done in [11], it is possible to make use of results obtained in the above-mentioned works for the estimate of the width of the considered process' spectral line. It is well known [9], that when the spectral density of frequency $\omega(f)$ fluctuations at $f = 0$ is zero, the formation of finite width of the line will not take place, and the problem will be reduced to the separation of the monochromatic line from noises. It may be shown, that the spectral density of fluctuations of the reduced difference of frequencies near zero is proportional to $f^{-2/3}$. This means that the spectral line will have a finite width, and for the determination of the difference of Doppler frequencies it is necessary to determine the maximum position in the spectrum $y(t)$.

One may use the following expression [10] for the estimate of the width of the spectral line:

$$\Delta f = \sqrt{B_v(0)}/2\pi, \tag{12}$$

where $B_v(\tau)$ is the correlation function of fluctuations of the difference of reduced frequencies.

Therefore, we are interested in the mean square of fluctuations of reduced frequencies' difference. We shall show, that knowing the two-dimensional spectrum of phases' reduced difference, we may find the two-dimensional spectrum of frequency difference assuming the frozen turbulence hypothesis. Let us make use of the decomposition of the locally-isotropic field of phase difference fluctuations:

$$\begin{gathered} S(\boldsymbol{r}) = S(0,0,0) + \int_{-\infty}^{\infty} \left(1 - e^{i\boldsymbol{\varkappa r}}\right) d\varphi(\boldsymbol{\varkappa}); \\ d\varphi(\varkappa)\, d\varphi^*(\varkappa') = \delta(\boldsymbol{\varkappa} - \boldsymbol{\varkappa}')\Phi_s(\boldsymbol{\varkappa}) d\boldsymbol{\varkappa} d\boldsymbol{\varkappa}', \end{gathered} \tag{13}$$

$\Phi_s(\varkappa)$ is the three-dimensional spectrum of the reduced difference of phases. Let us assume that the inhomogeneities are shifted by the wind in a transverse direction relative to the course (to the axis x) of the direction with a velocity $v_n$. Then the temporal fluctuations of phase difference may be considered with the aid of (13), provided we substitute in that expression y by $v_1 t$ and z by $v_3 t$:

$$\begin{gathered} S(x_1, v_2 t, v_3 t) = S(0) + \int_{-\infty}^{\infty} \left(1 - e^{i\varkappa_1 x_1} e^{i\varkappa_n v_n t}\right) d\varphi(\varkappa); \\ \varkappa_n = (\varkappa_2, \varkappa_3). \end{gathered} \tag{14}$$

Let us differentiate the fluctuations of phase difference in time :

$$\Delta\nu(x_1, v_2t, v_3t) = -\int_{-\infty}^{\infty} i(\varkappa_n v_n)\, e^{i\varkappa_1 x_1}\, e^{i\varkappa_n v_n t}\, d\varphi(\varkappa). \tag{15}$$

Returning to old variables, we shall obtain the decomposition of the fi d of fluctuations of the reduced phase difference :

$$\Delta\nu(r) = -\int_{-\infty}^{\infty} i(\varkappa_n v_n)\, e^{i\varkappa r}\, d\varphi(\varkappa). \tag{16}$$

the field $\Delta\nu(r)$ is uniform, but anisotropic. Let us determine the correlation function $\Delta\nu$:

$$B_\nu(\rho) = \overline{\Delta\nu(r)\Delta\nu^*(r-\rho)} = \int_{-\infty}^{\infty} (\varkappa_n v_n)^2 \Phi_s(\varkappa)\, e^{i\varkappa\rho}\, d\varkappa. \tag{17}$$

It follows from that expression that is the three-dimensional spectrum of fluctuations of the reduced frequencies' difference.

Let us introduce for consideration the two-dimensional spectrum of of fluctuations of the reduced Doppler frequency difference :

$$F_{\Delta\nu}(\varkappa_2, \varkappa_3, x_1) = \int_{-\infty}^{\infty} \Phi_{\Delta\nu}(\varkappa)\, e^{i\varkappa_1 x_1}\, d\varkappa_1. \tag{18}$$

In particular,

$$F_{\Delta\nu}(\varkappa_2, \varkappa_3, 0) = \int_{-\infty}^{\infty} \Phi_s(\varkappa)(\varkappa_n v_n)^2 d\varkappa_1 = (\varkappa_n v_n)^2 F_s(\varkappa_2, \varkappa_3, 0), \tag{19}$$

where $F_s(\varkappa_2, \varkappa_3, 0)$ is the two-dimensional spectrum of the reduced phase difference, $\overline{(\Delta\nu)^2} = B_\nu(0) = 2\int_{\infty}^{\infty} (\varkappa_n v_n)^2 F_s(\varkappa_2, \varkappa_3, 0)\, d\varkappa_2\, d\varkappa_3.$ (20)

. Let us find the time spectrum of fluctuations $\omega_o$ (f). Considering the correlation function $\Delta\gamma(r)$ in the plane x = L and assuming we shall consider the function to be the correlation function of fluctuations $\Delta\nu(r,t)$ [5]. Introducing the spectral density of the process

$$2\int_{-\infty}^{\infty} R_\nu(\tau)\, e^{-i\omega\tau}\, d\tau,$$

we find:

$$w_0(f) = 2\int_{-\infty}^{\infty} B_\nu(v_n\tau)\, e^{-i\omega\tau}\, d\tau = 2\int_{-\infty}^{\infty} (\varkappa_n v_n) F_s(\varkappa_2, \varkappa_3, 0)\, e^{i\varkappa_n v_n \tau}\, d\tau\, d\varkappa_n,$$

since

$$B_\nu(\rho) = \frac{1}{2\pi}\int_{-\infty}^{\infty} (\varkappa_n v_n)^2 e^{i\varkappa\rho} e^{-i\varkappa_1 x_1} F_s(\varkappa_2, \varkappa_3, x_1)\, d\varkappa\, dx_1 =$$
$$= \int_{-\infty}^{\infty} (\varkappa_n v_n)^2 F_s(\varkappa_2, \varkappa_3, \rho_1)\, e^{i\varkappa_n\rho}\, d\varkappa_n. \tag{21}$$

We finally obtain:

$$w_\nu(f) = 4\pi\int_{-\infty}^{\infty} \delta(\omega - v_n\varkappa_n)\, F_s(\varkappa_n, 0)\, (\varkappa_n v_n)^2\, d\varkappa_n. \tag{22}$$

For the sake of simplicity let us assume that $v_n$ is directed along the axis y. Then, integrating along $x_2$, we shall find:

$$w_\nu(f) = \frac{8\pi\omega^2}{v_n}\int_0^{\infty} F_s\left(\sqrt{\frac{\omega^2}{v_n^2} + \varkappa_3^2},\, 0\right) d\varkappa_3 = \omega^2 w_s(f),$$

for (see [5]):

$$\frac{8\pi}{v_n}\int_0^{\infty} F_s\left(\sqrt{\frac{\omega^2}{v_n^2} + \varkappa^2},\, 0\right) d\varkappa = w_s(f),$$

where $w_s(f)$ is the spectral density of fluctuations of the reduced difference of phases. This result may be fortold in advance, inasmuch as we are considering the derivative of the process with stationary increments. This derivative is a stationary process with a spectrum $w(f) = 4\pi f^2 w_s(f)$, where $w_s$ is the spectrum of the original process with stationary increments. Passing to polar coordinates, we find from the expression (20):

$$B_\nu(0) = 2\pi^2 v_n^2 \int_0^\infty F_s(\varkappa, 0)\, \varkappa^3 d\varkappa. \tag{23}$$

The estimate carried out according to formula (12) leads to the value of the effective width of the spectral line $\Delta f = 12$ c/s for the frequencies $f_{1,2} =$ 20 mc/s and 40 mc/s ($L = 200$ km, $v_n = 8$km sec$^{-1}$).

Therefore, the measured spectral line in the frequency $\nu_g$ may result rather strongly blurred. The precision of $\nu_g$ determination is thus directly linked with the accuracy in the determination of the position of the maximum in the spectrum $w_\nu(f)$ and is determined by the apparatus applied and the method of results of measurements' processing. Hence it follows that error in the $\nu_g$ measurement may be notably decreased in comparison with the quantity $\Delta f$. By the order of magnitude this is not in contradiction with the available experimental data [12].

A similar situation takes also place at measurement of $\bar{\theta}$. The values of the relation $\bar{\theta}/\sqrt{\overline{\Delta\theta^2}}$, brought out in Table 1, characterize the maximum fluctuations of $\theta$. Depending upon the method of results of measurement processing, the accuracy of $\theta$ determination may be overrated.

In conclusion we may note, that a more detailed consideration of the questions touched upon will provide the possibility of utilizing the above-examined experimental methods for the study of fluctuations in the ionosphere.

**** E N D ****

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